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Comments on noncommutative perturbative dynamics

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Abstract: We analyze further the IR singularities that appear in noncommutative field theories on $\mathcal{R}^d$. We argue that all IR singularities in nonplanar one loop diagrams may be interpreted as arising from the tree level exchanges of new light degrees of freedom, one coupling to each relevant operator. These exchanges are reminiscent of closed string exchanges in the double twist diagrams in open string theory. Some of these degrees of freedom are required to have propagators that are inverse linear or logarithmic. We suggest that these can be interpreted as free propagators in one or two extra dimensions respectively. We also calculate some of the IR singular terms appearing at two loops in noncommutative scalar field theories and find a complicated momentum dependence which is more difficult to interpret.

Keywords: Renormalization Regularization and Renormalons, Field Theories in Higher Dimensions, D-branes, Brane Dynamics in Gauge Theories.

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1. Introduction

In this note we continue the analysis of the perturbation expansion of field theories on noncommutative $\mathcal{R}^d$ [1]–[20]. We consider theories in various dimensions defined by an action

$$S = \int d^d x \, \mathrm{tr} \left(\frac{1}{2}(\partial\phi)^2 + \frac{1}{2}m^2\phi^2 + \sum_n \lambda_n \overbrace{\phi \star \phi \star \cdots \phi}^{n} \right), \tag{1.1}$$

where $\star$ is the noncommutative, associative star product defined by

$$f \star g(x) = e^{\frac{i}{2}\Theta^{\mu\nu}\partial^x_\mu \partial^y_\nu} f(x) g(y)|_{y=x} \,, \tag{1.2}$$

and Θ is a constant anticommuting noncommutativity matrix,

$$[x^\mu, x^\nu] = i\Theta^{\mu\nu} \,. \tag{1.3}$$

In [13], perturbative properties of these noncommutative scalar field theories were investigated through the explicit calculation of correlation functions. The $\star$-product form of the interactions leads to a momentum dependent phase associated with each vertex of a Feynman diagram. This phase is sensitive to the order of lines entering the vertex, so different orderings lead to diagrams with very different behavior. As was first demonstrated by Filk [1] , planar diagrams (with no crossings of lines) differ from the corresponding diagrams in the commutative theory only by external momentum dependent phase factors. These graphs lead to single trace terms like (1.1) in the effective action, including divergent terms which renormalize the bare action. The Feynman integrals for these graphs are the same as in the commutative case, resulting in the usual UV divergences which may be dealt with in the usual way by introducing counterterms.

Nonplanar diagrams contain internal momentum dependent phase factors associated with each crossing of lines in the graph. The oscillations of these phases serve to lessen any divergence, and may render an otherwise divergent graph finite, providing an effective cutoff $\Lambda_{\rm eff} = 1/\sqrt{\theta}$ in cases when internal lines cross (θ is a typical eigenvalue of $\Theta^{\mu\nu}$) or $\Lambda_{\rm eff} = 1/\sqrt{-p_\mu(\Theta^2)^{\mu\nu}p_\nu} \equiv 1/\sqrt{p \circ p}$ in cases where an external line with momentum p crosses an internal line. In the latter case, we see that the original UV divergence is replaced with an IR singularity, since taking $p \to 0$ results in $\Lambda_{\rm eff} \to \infty$.

This striking occurrence of IR singularities in massive theories suggests the presence of new light degrees of freedom. Indeed, an analysis of the one loop corrected propagator of ϕ reveals that in addition to the original pole at $p^2 \approx -m^2$, there is a new pole at $p^2 = \mathcal{O}(g^2)$. This new pole can be understood as arising from the high momentum modes of ϕ running in a loop. If we try to use a wilsonian effective action with a fixed cutoff Λ, these modes are absent, and indeed, we find that the $p \to 0$ limit is not singular, since the effective cutoff $\Lambda_{\rm eff}$ is replaced by the cutoff Λ when $p \circ p < 1/\Lambda^2$. Thus the $p \to 0$ and $\Lambda \to \infty$ limits do not commute. In order to write a wilsonian effective action that does correctly describe the low momentum behavior of the theory, it is necessary to introduce new fields into the action which represent the light degrees of freedom. In [13], it was shown that the quadratic IR divergences in the two point functions of ϕ^4 in four dimensions or ϕ^3 in six dimensions can be reproduced by adding a field χ_0 with action of the form

$$S_{\chi_0} = \int d^dx \; g\chi_0 \,\mathrm{tr}(\phi) + \frac{1}{2}\partial\chi_0 \circ \partial\chi_0 + \frac{1}{2}\Lambda^2(\partial \circ \partial\chi_0)^2 \,. \tag{1.4}$$

With this action, the quadratic IR singularity in the two point function of ϕ is reproduced by a diagram in which ϕ turns into χ_0 and back into ϕ.

It should be stressed that in lorentzian signature spacetime with $\Theta^{0i} = 0$ the field χ_0 is not dynamical. It is a Lagrange multiplier [13]. Yet, it does lead to long range correlations. Even though it is not a propagating field in this case, we will loosely refer to it as a particle.

These effects are very reminiscent of channel duality in string theory [13]. There, high momentum open strings running in a loop have a dual interpretation as the exchange of a light closed string. By this analogy, we may associate the field ϕ with the modes of open strings, while χ_0 describes a closed string mode. We thus see that noncommutative field theories are interesting toy models of open string theories. Other evidence to the stringy nature of these theories is their T-duality behavior when they are compactified on tori and their large Θ behavior [13].

Clearly, the appearance of these closed string modes is a generic phenomenon occurring whenever the commutative theory exhibits UV divergences. It is surprising because the zero slope limit of [21] is supposed to decouple all the higher open string modes of the string as well as the closed string modes. In hindsight this phenomenon is perhaps somewhat less surprising. The fields living on a brane are modes of open strings. The parameters in this theory are the zero momentum modes of the closed string background in which the brane is embedded. If the theory on the brane is not conformal, the renormalization group in the theory on the brane changes the values of these parameters. Therefore, it is typical for the zero momentum modes of the closed strings not to decouple. We now see that in the noncommutative theories, the nonzero modes also fail to decouple.

Given this understanding of the quadratic IR singularities as poles of a light particle, it is interesting to ask whether we can find a similar interpretation for the inverse linear and logarithmic IR singularities that appear. These occur wherever linear or logarithmic UV divergences appear in the commutative theories, including two point functions and higher point interactions. It is the goal of this paper to provide some further understanding of these logarithmic and inverse linear IR singularities.

In the next section, we determine the complete set of IR singularities in the low energy effective action that arise from one loop graphs in various scalar theories. We show that they may be reproduced by including a set of χ fields coupling to each relevant operator of the theory, if we allow some of the fields χ to have propagators which behave like $\ln(1/p\circ p)$ or $(p\circ p)^{-1/2}$. In section 3, we point out that these propagators arise naturally if the χ fields are actually free particles in extra dimensions, coupling to ϕs that live on a brane of codimension two for logarithmic singularities or codimension one for $(p \circ p)^{-1/2}$ singularities. This is natural given the analogy with string theory, since we associate the ϕ particles with open string modes which live on a brane, while the χs are closed strings which should be free to propagate in the bulk. In section 4, we consider higher loop graphs in scalar field theory, and find IR singularities with more complicated momentum dependence that are more difficult to interpret.

2. Low energy one loop effective action

In this section, we write down the complete set of IR singular terms in the one loop 1PI effective actions of various scalar theories. We will take ϕ to be an $N \times N$ matrix since it is useful to see how the indices are contracted in the various terms.

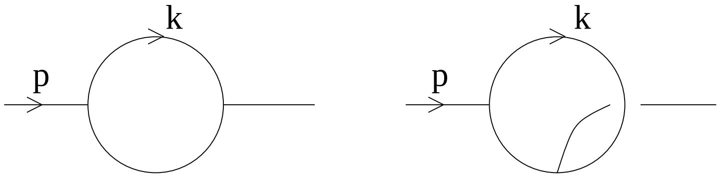

Figure 1: Planar and nonplanar contributions to the one loop quadratic effective action in ϕ^3 theory in four dimensions.

In particular, it turns out that all IR singular terms take the form

$$\mathrm{tr}(\mathcal{O}_1(p))\,\mathrm{tr}(\mathcal{O}_2(-p))f(p)\,, \tag{2.1}$$

where the $\mathcal{O}$s are operators built out of ϕ, and $f(p)$ diverges quadratically, linearly, or logarithmically as $p \to 0$. We show that any such term may be understood as arising from the exchange of a single scalar particle which couples separately to $\mathrm{tr}(\mathcal{O}_1)$ and $\mathrm{tr}(\mathcal{O}_2)$ and which has a propagator $f(p)$.

2.1 ϕ^3 theory in $d = 4$

We begin by considering the simple example of ϕ^3 theory in four dimensions

$$S = \int d^4x\;\mathrm{tr}\left(\frac{1}{2}(\partial\phi)^2 + \frac{1}{2}m^2\phi^2 + \frac{g}{3!}\phi\star\phi\star\phi\right). \tag{2.2}$$

The commutative theory is superrenormalizable, and the only UV divergences are logarithmic divergences in the one loop contributions to the 1PI effective action. These come from the planar and nonplanar diagrams shown in figure 1 which contribute respectively to $\mathrm{tr}(\phi^2)$ and $\mathrm{tr}(\phi)\,\mathrm{tr}(\phi)$ terms in the effective action.

In the noncommutative theory, the planar diagram is unchanged, while the nonplanar diagram becomes finite, cutoff by $\Lambda^2_{\mathrm{eff}} = 1/p\circ p$. Combining the two contributions, we find that the one loop quadratic effective action (at finite cutoff) is

$$\begin{aligned} S_{\mathrm{eff}} = (2\pi)^4 \int d^4p\,\frac{N}{2}\,\mathrm{tr}(\phi(p)\phi(-p))(p^2+M^2) - {}\\ -\frac{1}{2}\,\mathrm{tr}(\phi(p))\,\mathrm{tr}(\phi(-p))\frac{g^2}{64\pi^2}\ln\left(\frac{1}{M^2(p\circ p+1/\Lambda^2)}\right) + \cdots\,, \end{aligned} \tag{2.3}$$

where M is the planar renormalized mass, corrected at one loop by the planar diagram in figure 1 plus a counterterm graph.

The second term in (2.3) arises from the nonplanar diagram and contains a logarithmic IR singularity for $\Lambda = \infty$ but not at finite cutoff. Thus, as in [13], the $\Lambda \to \infty$ and $p \to 0$ limits do not commute. In order to reproduce the correct low momentum behavior in a wilsonian action, we must introduce a new field.

As for the case of theories with quadratic divergences considered in [13], we introduce a new field χ which couples to $\mathrm{tr}(\phi)$

$$\int d^dx\; g\chi(x)\,\mathrm{tr}(\phi(x))\,. \tag{2.4}$$

Now, suppose that χ has a propagator given by

$$\langle\chi(p)\chi(-p)\rangle = f(p)\,. \tag{2.5}$$

Then upon integrating out χ, the quadratic effective action for ϕ receives a contribution

$$S = (2\pi)^d \int d^dp \frac{1}{2}\,\mathrm{tr}(\phi(p))\,\mathrm{tr}(\phi(-p))\left(-g^2 f(p)\right). \tag{2.6}$$

Thus, we see that the logarithmic singularity in (2.3) may be reproduced by including a coupling (2.4) in the wilsonian effective action, if χ has a momentum space propagator

$$f(p) = \frac{1}{64\pi^2}\ln\left(\frac{p\circ p + 1/\Lambda^2}{p\circ p}\right). \tag{2.7}$$

The numerator in the logarithm has been chosen to cancel the incorrectly cutoff logarithm coming from the nonplanar ϕ loop in the cutoff theory.

In this theory, there are no additional singularities at higher loops, so this single new field is enough to reproduce all IR singularities. In section 3, we will give a possible interpretation of this logarithmic propagator, but first we turn to more complicated examples.

2.2 ϕ^4 theory in $d = 4$

As a second example, we consider ϕ^4 theory in four dimensions

$$S = \int d^4x\;\mathrm{tr}\left(\frac{1}{2}(\partial\phi)^2 + \frac{1}{2}m^2\phi^2 + \frac{g^2}{4!}\phi\star\phi\star\phi\star\phi\right). \tag{2.8}$$

The effective action for the case where ϕ is not a matrix ($N = 1$) was computed in [13] (for low momenta),

$$\begin{aligned}
S_{\mathrm{eff}} = (2\pi)^4 \int d^4p \frac{1}{2}\phi(p)\phi(-p)\Bigg(p^2 + M^2 + \frac{g^2}{96\pi^2(p\circ p + 1/\Lambda^2)} -& \\
-\frac{g^2M^2}{96\pi^2}\ln\left(\frac{1}{M^2(p\circ p+1/\Lambda^2)}\right) + \cdots\Bigg) &+ \\
+(2\pi)^4\int d^4p_i \frac{1}{4!}\phi(p_1)\phi(p_2)\phi(p_3)\phi(p_4)\delta\left(\sum p_i\right) \times& \\
\times\Bigg(g^2 - \frac{g^4}{3\cdot 2^5\pi^2}\sum_i \ln\left(\frac{1}{M^2(p_i\circ p_i + 1/\Lambda^2)}\right) -& \\
-\frac{g^4}{3\cdot 2^6\pi^2}\sum_{i<j}\ln\left(\frac{1}{M^2((p_i+p_j)\circ(p_i+p_j)+1/\Lambda^2)}\right) + \cdots\Bigg)&.
\end{aligned} \tag{2.9}$$

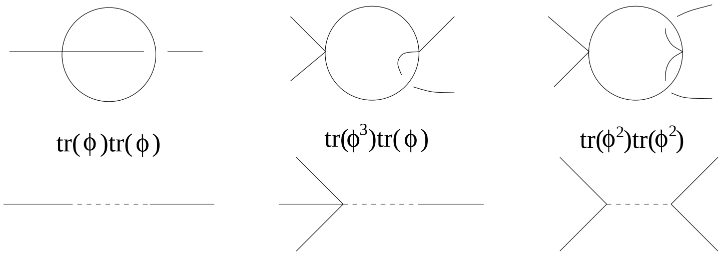

Figure 2: Examples of nonplanar diagrams in ϕ^4 in $d = 4$ contributing IR singularities for $\Lambda \to \infty$ and χ exchange diagrams that reproduce the singularities at finite cutoff.

In this case, the quadratic effective action has both quadratic and logarithmic IR singularities for $\Lambda = \infty$, while the quartic term has two types of logarithmic singularities. In [13], it was shown that the $1/p \circ p$ term in the $\Lambda \to \infty$ quadratic effective action is reproduced by a wilsonian effective action which includes a χ_0 field coupling to ϕ with action of the form (1.4) .

We now focus on the terms containing logarithmic singularities. It is illustrative to generalize to the case of arbitrary N and write them as

$$\begin{aligned} &\int d^4x d^4y \left\{ -g^2 M^2 \operatorname{tr}(\phi(x)) \operatorname{tr}(\phi(y)) - \right. \\ &\left. -\frac{g^4}{3} \operatorname{tr}(\phi(x)) \operatorname{tr}(\phi^3(y)) - \frac{g^4}{4} \operatorname{tr}(\phi^2(x)) \operatorname{tr}(\phi^2(y)) \right\} \times \\ &\times \frac{1}{3 \cdot 2^6 \pi^2} \int \frac{d^4p}{(2\pi)^4} e^{ip\cdot(x-y)} \ln\left(\frac{1}{M^2(p \circ p + 1/\Lambda^2)}\right) = \\ &= \int d^4x d^4y \sum g^{m+n} M^{4-m-n} \gamma_{mn} \operatorname{tr}(\phi^m(x)) \operatorname{tr}(\phi^n(y)) \Delta(x-y)\,, \end{aligned} \tag{2.10}$$

where

$$\Delta(x-y) = \int \frac{d^4p}{(2\pi)^4} e^{ip\cdot(x-y)} \ln\left(\frac{1}{M^2(p \circ p + 1/\Lambda^2)}\right), \tag{2.11}$$

and γ_{mn} are numerical constants that may be read off from (2.10). In this form it is clear that the complete set of logarithmic IR singularities in the $\Lambda \to \infty$ one loop effective action may be reproduced with a finite cutoff wilsonian action by including χ fields with couplings

$$\int d^4x \sum_{n=1}^{3} g^n M^{2-n} \chi_n(x) \operatorname{tr}(\phi^n(x)) \tag{2.12}$$

and logarithmic propagators

$$\langle \chi_m(p) \chi_n(-p) \rangle = -2\gamma_{mn} \ln\left(\frac{p \circ p + 1/\Lambda^2}{p \circ p}\right). \tag{2.13}$$

In this way, the $\Lambda = \infty$ IR singularity in each term of (2.10) is reproduced at finite cutoff by the exchange of a single scalar particle, as shown in figure 2.

2.3 ϕ^3 theory in $d=6$

As a final example, we consider ϕ^3 theory in six dimensions,

$$S=\int d^6x\,\mathrm{tr}\left(\frac{1}{2}(\partial\phi)^2+\frac{1}{2}m^2\phi^2+\frac{g}{3!}\phi\star\phi\star\phi\right). \tag{2.14}$$

Here, the one loop effective action (for low momenta) was computed in [13] for $N=1$,

$$\begin{aligned}S_{\mathrm{eff}}=(2\pi)^6\int d^6p\frac{1}{2}\phi(p)\phi(-p)\left(p^2+M^2-\frac{g^2}{2^8\pi^3(p\circ p+1/\Lambda^2)}+\right.\\ \left.+\frac{g^2}{3\cdot 2^9\pi^3}(p^2+6M^2)\ln\left(\frac{1}{M^2(p\circ p+1/\Lambda^2)}\right)+\cdots\right)+\\ +(2\pi)^6\int d^6p_i\frac{1}{3!}\phi(p_1)\phi(p_2)\phi(p_3)\delta\left(\sum p_i\right)\times\\ \times\left\{g+\frac{g^3}{2^9\pi^3}\sum_i\ln\left(\frac{1}{M^2(p_i\circ p_i+1/\Lambda^2)}\right)\right\}.\end{aligned} \tag{2.15}$$

The IR singularities in this action at $\Lambda=\infty$ are similar to those of the ϕ^4 theory, but now we have a $p^2\ln(1/M^2p\circ p)$ term in the quadratic effective action. We may rewrite the terms with logarithmic singularities for arbitrary N as

$$\begin{aligned}\int d^6x d^6y\left\{gM\,\mathrm{tr}(\phi(x))\left(gM\,\mathrm{tr}(\phi(y))-\frac{g}{6M}\,\mathrm{tr}(\partial^2\phi(y))+\frac{g^2}{2M}\,\mathrm{tr}(\phi^2(y))\right)\right\}\times\\ \times\frac{1}{512\pi^3}\int\frac{d^6p}{(2\pi)^6}e^{ip\cdot(x-y)}\ln\left(\frac{1}{M^2p\circ p}\right).\end{aligned} \tag{2.16}$$

These may be reproduced by introducing χ fields with couplings

$$\int d^6xgM\chi_1(x)\,\mathrm{tr}(\phi(x))+\chi_2(x)\left(gM\,\mathrm{tr}(\phi(x))+\frac{g^2}{2M}\,\mathrm{tr}(\phi^2(x))-\frac{g}{6M}\,\mathrm{tr}(\partial^2\phi(x))\right) \tag{2.17}$$

and propagators

$$\begin{aligned}\langle\chi_1(p)\chi_2(-p)\rangle &= -\frac{1}{512\pi^3}\ln\left(\frac{p\circ p+1/\Lambda^2}{p\circ p}\right)\\ \langle\chi_1(p)\chi_1(-p)\rangle &= \langle\chi_2(p)\chi_2(-p)\rangle=0\,.\end{aligned} \tag{2.18}$$

By a linear redefinition of the χs, we may simplify the couplings to

$$\int d^6xgM\hat{\chi}_1(x)\,\mathrm{tr}(\phi(x))+\hat{\chi}_2(x)\left(\frac{g^2}{2M}\,\mathrm{tr}(\phi^2(x))-\frac{g}{6M}\,\mathrm{tr}(\partial^2\phi)\right), \tag{2.19}$$

where $\hat{\chi}_1=\chi_1+\chi_2$ and $\hat{\chi}_2=\chi_2$.

2.4 General procedure

The three examples we have considered lead us to a general procedure for introducing χ fields in the wilsonian effective action in order to reproduce logarithmic singularities in the one loop effective action. For a general scalar theory, logarithmic IR singular terms in the effective action arising from one loop non-planar diagrams take the form

$$\sum_{m,n} \int d^4x d^4y \frac{1}{2} \mathcal{O}_m(\phi(x)) \mathcal{O}_n(\phi(y)) \gamma_{mn} \int \frac{d^dp}{(2\pi)^d} e^{ip\cdot(x-y)} \ln\left(\frac{1}{m^2(p\circ p + 1/\Lambda^2)}\right). \tag{2.20}$$

Here $\{\mathcal{O}_n(\phi(x))\}$ is some basis for the set of relevant local operators (such as $\mathrm{tr}(\phi^m)$, $\mathrm{tr}(\partial^2\phi)$, etc...). γ_{mn} is a "metric" on the space of operators, which we may take to be a matrix of numerical constants by assuming that all masses and coupling constants are included in the $\mathcal{O}$s. Note that terms in the effective action at higher loops will involve products of more than two operators, however the one loop terms may always be written in this form. We now introduce a χ field coupling to each $\mathcal{O}$,

$$S_{\chi\phi} = \int d^dx \chi_n(x) \mathcal{O}_n(\phi(x)), \tag{2.21}$$

and assume that the fields χ have propagators

$$\langle \chi_m(p)\chi_n(-p)\rangle = -\gamma_{mn} \ln\left(\frac{p\circ p + 1/\Lambda^2}{p\circ p}\right), \tag{2.22}$$

which could arise, for example, from the nonlocal quadratic action

$$S_{\chi\chi} = -\int d^dp \frac{1}{2} \chi_m(p)\chi_n(-p)\gamma^{mn} \left[\ln\left(\frac{p\circ p + 1/\Lambda^2}{p\circ p}\right)\right]^{-1}. \tag{2.23}$$

Here, γ^{mn} is the inverse[1] of γ_{mn}.

2.5 Linear divergences in ϕ^4 in three dimensions

Before closing this section, we note that linear IR singularities at one loop may be understood in a similar manner. For example, the commutative ϕ^4 theory in $d = 3$ has a linear divergence in the two point function at one loop. In the noncommutative theory, this leads to a term

$$S_{IR} = \int d^3p \, \frac{1}{2} \, \mathrm{tr}(\phi(p)) \, \mathrm{tr}(\phi(-p)) \frac{\pi^2}{6(p\circ p + 1/\Lambda^2)^{1/2}} \tag{2.24}$$

[1] If γ is not invertible, we have simply introduced too many χs. In this case, we choose a new basis $\{\hat{\mathcal{O}}_n\}$ of operators such that some of the basis elements do not appear in (2.20). The submatrix $\hat{\gamma}$ of γ corresponding to the $\hat{\mathcal{O}}$s which do appear will then be invertible, and we may introduce χs as above coupling to this smaller set of operators. The kinetic term (2.23) for the χs is then well defined, since we replace γ with $\hat{\gamma}$.

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in the quadratic effective action which has a $1/|p|$ singularity for $\Lambda = \infty$. In order to reproduce this singularity, we again introduce a χ field coupling to $\mathrm{tr}(\phi)$ but this time we need a propagator

$$\langle \chi(p)\chi(-p)\rangle \propto \frac{1}{(p\circ p)^{1/2}} - \frac{1}{(p\circ p + 1/\Lambda^2)^{1/2}}\,. \tag{2.25}$$

3. What can give logarithmic and inverse linear propagators?

In the previous section we showed that the singular IR behavior of the one loop effective actions for various scalar field theories can be reproduced by a wilsonian action which includes new particles χ_i coupling linearly to various operators built from of ϕ, assuming that the fields χ have logarithmic (or inverse linear) propagators. We would now like to understand what dynamics could give rise to these propagators.

3.1 Spectral representation of propagators

As a first step, it is useful to rewrite the propagators in a spectral representation. We introduce a parameter[2] α' with dimensions of squared length and a metric $g^{\mu\nu} = -(\Theta^2)^{\mu\nu}/(\alpha')^2$ then rewrite the propagator as

$$\Delta(p) = \int dm^2 \rho(m^2) \frac{1}{p\circ p/(\alpha')^2 + m^2}\,. \tag{3.1}$$

The logarithmic propagators

$$\Delta(p) = \ln\left(\frac{p\circ p + 1/\Lambda^2}{p\circ p}\right) \tag{3.2}$$

are reproduced with the spectral density

$$\rho(m^2) = \begin{cases} 1 & m^2 \le \dfrac{1}{(\alpha'\Lambda)^2} \\ 0 & m^2 > \dfrac{1}{(\alpha'\Lambda)^2} \end{cases} \tag{3.3}$$

suggesting a continuum of states with m^2 uniformly distributed between 0 and a cutoff $1/(\alpha'\Lambda)^2$. Thus, we may replace χ with a continuum of states ψ_m which have ordinary propagators

$$\langle \psi_m(p)\psi_m(-p)\rangle \propto \frac{1}{p\circ p/(\alpha')^2 + m^2}\,, \tag{3.4}$$

[2]We call the constant α' since for noncommutative field theories arising from string theory, the metric $g^{\mu\nu} = -(\Theta^2)^{\mu\nu}/(\alpha')^2$ is exactly the closed string metric in the zero slope limit of [21] (the open string metric is $G_{\mu\nu} = \delta_{\mu\nu}$) when α' has its usual meaning.

which couple to ϕ (or some $\mathcal{O}(\phi)$) in a way that is independent of m,

$$S_{\psi\phi} = \int^{1/(\alpha'\Lambda)^2} dm^2 \int d^d x \psi_m(x)\phi(x)\,. \tag{3.5}$$

Note that for $\Lambda \to \infty$, the ψs completely decouple, leaving the original ϕ theory, as desired.

The $\Delta(p) = 1/(p \circ p)^{1/2}$ propagators, are reproduced by a spectral density[3]

$$\rho(m^2) = \begin{cases} \dfrac{1}{m\pi\alpha'} & m^2 \lesssim \dfrac{4}{(\Lambda\pi\alpha')^2} \\ 0 & m^2 \gtrsim \dfrac{4}{(\Lambda\pi\alpha')^2}\,. \end{cases} \tag{3.6}$$

As above, we may interpret this as a continuum of states ψ_m coupling to ϕ, this time with density $1/(\alpha' m)$ up to a cutoff $m^2 \approx 1/(\alpha'\Lambda)^2$.

3.2 χs as degrees of freedom in extra dimensions

A very simple possibility for the interpretation of the continuum of degrees of freedom ψ_m is that they are the transverse momentum modes of a particle ψ which propagates freely in more dimensions. The continuous parameter m is related to the momentum q in these new dimensions through $m^2 = -q^2$. That is, we imagine that the d-dimensional space in which the ϕ quanta propagate is a flat d-dimensional brane residing in a $d+n$ dimensional space. The ψ particles propagate freely in this space but couple to the ϕ particles on the brane (located at $x_\perp = 0$). We choose the metric seen by the ψ particles to be $g^{\mu\nu} = -(\Theta^2)^{\mu\nu}/(\alpha')^2$ in the brane directions and $\delta^{\mu\nu}$ in the transverse directions.

By choosing the number of extra dimensions to be one or two, we can precisely reproduce the spectral densities (3.6) or (3.3) corresponding to inverse linear or logarithmic propagators respectively.

Explicitly, with two extra dimensions we have

$$\begin{aligned} \langle \psi(p, x_\perp{=}0)\psi(-p, x_\perp{=}0)\rangle &= \int^{1/\alpha'\Lambda} \frac{d^2q}{(2\pi)^2} \frac{1}{p\circ p/(\alpha')^2 + q^2} \\ &= \int^{1/(\alpha'\Lambda)^2} \frac{dm^2}{4\pi} \frac{1}{p\circ p/(\alpha')^2 + m^2} \\ &= \frac{1}{4\pi} \ln\left(\frac{p\circ p + 1/\Lambda^2}{p\circ p}\right), \end{aligned} \tag{3.7}$$

giving exactly the desired form of the logarithmic propagators. Note that we impose a cutoff $1/(\Lambda\alpha')^2$ on q^2 to reproduce the cutoff in the spectral function (3.6). This cutoff on transverse momenta *decreases* to zero as $\Lambda \to \infty$, but this provides the desired decoupling of ψ in this limit.

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[3] The symbols $\gtrsim$ and $\lesssim$ are used because the second term in (2.25) is regularization dependent and its precise functional form is not reproduced by choosing a sharp cutoff here, though the behavior for $p \to 0$ at fixed Λ and for $\Lambda \to \infty$ is the same.

With one extra dimension, we find

$$\begin{aligned}
\langle\psi(p,x_\perp{=}0)\psi(-p,x_\perp{=}0)\rangle &= \int^{2/\Lambda\pi\alpha'}\frac{dq}{2\pi}\frac{1}{p\circ p/(\alpha')^2+q^2}\\
&= \int^{4/(\Lambda\pi\alpha')^2} dm^2\frac{1}{2\pi m}\frac{1}{p\circ p/(\alpha')^2+m^2}\\
&= \frac{\alpha'}{2(p\circ p)^{1/2}}-\frac{\alpha'}{\pi(p\circ p)^{1/2}}\tan^{-1}\left(\frac{\Lambda\pi(p\circ p)^{1/2}}{2}\right)
\end{aligned} \quad (3.8)$$

thus giving a spectral density of $1/m$ and reproducing the correct $p\to 0$ behavior of the inverse linear propagators (2.25) above (as discussed in footnote 3, the difference in functional form between the second terms here and in (2.25) is a consequence of the choice of regularization scheme).

Actually, it is possible to see more directly that the theory with free ψ particles in two extra dimensions coupling linearly to ϕs on the brane is precisely equivalent to a theory with particles χ that live on the brane and have logarithmic propagators. Noting that it is $\psi(x,x_\perp=0)$ that couples to ϕ, we define $\chi(x)=\psi(x,x_\perp=0)$ and rewrite the ψ action using a Lagrange multiplier $\lambda(x)$ as

$$\begin{aligned}
&e^{-\int d^dx\phi(x)\psi(x,x_\perp=0)-\int d^dxd^2x_\perp\frac{1}{2}(\partial\psi)^2} =\\
&\quad= \int[d\lambda][d\chi]e^{-\int d^dx\,[\phi(x)\chi(x)+i\lambda(x)[\chi(x)-\psi(x,x_\perp=0)]]-\int d^dxd^2x_\perp\frac{1}{2}(\partial\psi)^2} \qquad (3.9)\\
&\quad= \int[d\lambda][d\chi]e^{-\int d^dp[\phi(p)\chi(-p)+i\lambda(-p)\chi(p)]-\int d^dpd^2q\left[\frac{1}{2}\psi(-p,-q)(p\circ p+q^2)\psi(p,q)-i\lambda(-p)\psi(p,q)\right]}\,.
\end{aligned}$$

We may then integrate out ψ directly from this action, leaving

$$e^{-\int d^dp\left[\phi(p)\chi(-p)+i\lambda(p)\chi(-p)+\frac{1}{2}\lambda(p)\lambda(-p)\int\frac{d^2q}{p\circ p+q^2}\right]}\,. \qquad (3.10)$$

Finally, integrating out the Lagrange multiplier λ gives

$$e^{-\int d^dp\,\left[\phi(p)\chi(-p)+\frac{1}{2}\chi(-p)\chi(p)\ln^{-1}\left(\frac{p\circ p+1/\Lambda^2}{p\circ p}\right)\right]} \qquad (3.11)$$

as desired.

In the general case, we find that the nonlocal quadratic action (2.23) derived above may be replaced by an ordinary, local higher dimensional kinetic term

$$\hat{S}_{\psi\psi} = -\frac{1}{8\pi}\int d^{d+2}x\frac{1}{2}g^{\mu\nu}\partial_\mu\psi_m\partial_\nu\psi_n\gamma^{mn}, \qquad (3.12)$$

where $g^{\mu\nu}$ is taken to be $-(\Theta^2)^{\mu\nu}/(\alpha')^2$ in the original d directions and $\delta^{\mu\nu}$ in the transverse directions.

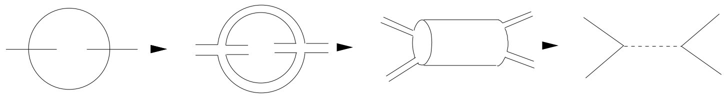

Figure 3: Channel duality in string theory provides a natural understanding of the correspondence between nonplanar one loop diagrams and tree level χ exchange diagrams.

As an example, the logarithmic terms in the ϕ^3 theory in six dimensions may be reproduced using

$$\int d^6x \, \frac{gm}{3 \cdot 32\pi}\phi(x)\psi(x, x_\perp{=}0) + \left(\frac{3g^2}{16\pi m}\phi^2(x) - \frac{g}{192\pi m}\partial^2\phi(x)\right)\hat{\psi}(x, x_\perp{=}0) + \\ + \int d^8x \frac{1}{2}g^{\mu\nu}\partial_\mu\hat{\psi}\partial_\nu\hat{\psi} - \frac{1}{12}g^{\mu\nu}\partial_\mu\psi\partial_\nu\hat{\psi}\,. \tag{3.13}$$

3.3 String theory analogy

The suggestion that certain high momentum degrees of freedom in ϕ are dual to fields which propagate in extra dimensions fits rather nicely with the analogy that associates ϕs with open strings and χs or ψs with closed strings. In noncommutative gauge theories that arise as limits of string theory, the fields (ϕs) in terms of which the theory is defined are modes of open strings living on a D-brane. In this case, there is a physical bulk in which closed strings propagate, and the low energy closed string modes are related to high energy modes of open strings by channel duality. In particular, a nonplanar one loop diagram of the type we have studied is topologically equivalent to a string diagram in which a number of open strings become a closed string which then turns back into open strings. The regime in which the open string loop has very high momenta may be viewed equivalently as the exchange of a very low momentum closed string, free to propagate in the bulk. This fits very well with our interpretation that the IR singularities of nonplanar one loop diagrams should be reproduced by tree level exchanges of a particle ψ (see figure 3). The connection between ψ and closed strings is further strengthened by the fact that the ψs do not carry any matrix indices, and as noted above, the ψs propagate in a metric which is precisely the closed string metric identified in [21].

3.4 Interpretations of the extra dimensions

Despite the many similarities between the ψ fields and closed strings, the calculations we have presented so far do not really indicate whether the extra dimensions in which the ψs propagate are truly physical. There seem to be three logical possibilities:

1. The first possibility is that the "extra dimensions" are simply a mathematical convenience — a simple way to state that χ has a logarithmic propagator.

2. At the other extreme is the possibility that the extra dimensions are real and the ψs are propagating fields living in these dimensions whose restrictions to the d dimensional space are the χ fields.

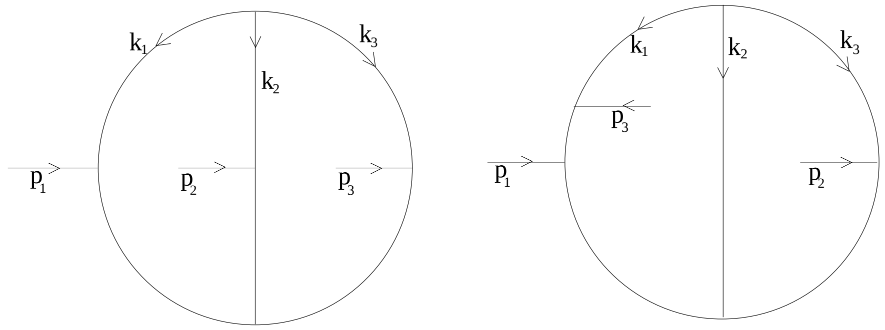

Figure 4: The two contributions to the $(\mathrm{tr}\,\phi)^3$ term in the effective action at two loops.

3. The third possibility is a compromise between these two: in situations where the noncommutative field theory is a limit of string theory, the extra dimensions are really the bulk dimensions transverse to the brane on which the ϕ fields live and the extra dimensions are physical. Otherwise, they are only a mathematical convenience.

We note that in the case of noncommutative field theories arising from string theory, the number of dimensions transverse to the brane is typically larger than two, so the propagator of a single massless bulk field between points on the brane is nonsingular. However, in these theories there is no reason to expect that only a single closed string mode ψ contributes. Rather, the spectral function required to reproduce IR singularities would presumably arise from the combination of extra dimensions and the density of closed string states which are allowed to couple to the worldvolume field of interest.[4]

4. Higher loops

4.1 The three point function of ϕ^3 at two loops

In this section, we explore the infrared singularities appearing in $\Gamma^{(3)}$ at two loops for the noncommutative ϕ^3 theory in six dimensions. In particular, we consider the two diagrams of figure 4 which give the leading contribution to the $(\mathrm{tr}\,\phi)^3$ term in the effective action.

These diagrams have the property that each external line connects to a different index loop in double line notation. The remaining two loop diagrams with three external lines give subleading contributions either to $\mathrm{tr}(\phi)\,\mathrm{tr}(\phi^2)$ or to $\mathrm{tr}(\phi^3)$ terms.

The first diagram contributes a term to the effective action

$$\int d^6p_1 d^6p_2 d^6p_3 \delta(p_1+p_2+p_3)\phi(p_1)\phi(p_2)\phi(p_3)V_1(p_1,p_2,p_3)\,, \tag{4.1}$$

[4] We thank Lenny Susskind and Igor Klebanov for helpful discussions on this point.

where

$$V_1(p_1,p_2,p_3) = \frac{g^5}{3\cdot 2^{11}\pi^6} \times \tag{4.2}$$
$$\times \int \frac{d^6k_1 d^6k_2 d^6k_3 \delta(k_1+k_2+k_3) e^{ik_1\times p_2 - ik_3\times p_3}}{(k_1^2+m^2)((k_1+p_1)^2+m^2)(k_2^2+m^2)((k_2+p_2)^2+m^2)(k_3^2+m^2)((k_3+p_3)^2+m^2)},$$

and $k \times p \equiv k_\mu \theta^{\mu\nu} p_\nu$. The external momenta appearing in the denominator do not contribute to the infrared singular terms, since by Taylor expanding the denominators in momenta, we find that the subleading terms are nonsingular for $p \to 0$. To evaluate the leading terms, we rewrite the delta function as $\int e^{iy\cdot(k_1+k_2+k_3)}$ to obtain

$$V_1(p_1,p_2,p_3) = \frac{g^5}{3\cdot 2^{11}\pi^6} \int \frac{d^6y}{(2\pi)^6} \prod_{i=1}^{3} \int \frac{d^6k_i e^{ik_i\cdot x_i}}{(k_i^2+m^2)^2}, \tag{4.3}$$

where $x_1 = y + \theta p_2$, $x_2 = y$, $x_3 = y - \theta p_3$. The IR singular terms come from the region of small y, so we need the behavior of the k integrals for small x.

In this regime, we have

$$\int \frac{d^6k_i e^{ik_i\cdot x_i}}{(k^2+m^2)^2} = \frac{4\pi^3}{x^2} - m^2\pi^3 \ln\left(\frac{1}{x^2m^2}\right) + \cdots. \tag{4.4}$$

Only the leading term contributes to the IR singular pieces, so our Feynman integral becomes

$$V_1(p_1,p_2,p_3) = \frac{g^5\pi^3}{3\cdot 2^5} \int^\Lambda \frac{d^6y}{(2\pi)^6} \frac{1}{y^2(y+\theta p_2)^2(y-\theta p_3)^2}. \tag{4.5}$$

The cutoff Λ has been included because our approximations are valid only for small y, but this is the only part of the integral that gives the IR singular terms of interest. This integral clearly has a logarithmic singularity as all momenta go to zero, but is finite, if any one momentum is scaled to zero. By a careful analysis of the integral (4.5), it may be shown that for small momenta, we have

$$V_1(p_1,p_2,p_3) = \frac{g^5}{3\cdot 2^{12}} \ln\left(\frac{1}{p_1\circ p_1 + p_2\circ p_2 + p_3\circ p_3}\right) + \text{regular terms}. \tag{4.6}$$

We now turn to the second diagram of figure 3. This contributes a term to the effective action

$$\int d^6p_1 d^6p_2 d^6p_3 \delta(p_1+p_2+p_3)\phi(p_1)\phi(p_2)\phi(p_3)V_2(p_1,p_2,p_3), \tag{4.7}$$

where

$$V_2(p_1,p_2,p_3) = \frac{g^5}{2^{10}\pi^6} \int \frac{d^6k_1 d^6k_2 d^6k_3 \delta(k_1+k_2+k_3) e^{ik_3\times p_2 - ik_1\times p_3}}{(k_3^2+m^2)((k_3+p_2)^2+m^2)(k_1^2+m^2)((k_1+p_3)^2+m^2)} \times$$
$$\times \frac{1}{((k_1+p_3+p_1)^2+m^2)(k_2^2+m^2)}. \tag{4.8}$$

As above, the momenta in the denominator do not affect the IR singular terms and we may rewrite the integral as

$$V_2(p_1,p_2,p_3) = \frac{g^5}{2^{10}\pi^6}\int \frac{d^6y}{(2\pi)^6}\int \frac{d^6k_1 d^6k_2 d^6k_3 e^{ik_i\cdot x_i}}{(k_1^2+m^2)^3(k_2^2+m^2)(k_3^2+m^2)^2}\,, \tag{4.9}$$

with $x_1 = y - \theta p_3$, $x_2 = y$, $x_3 = y + \theta p_2$. The IR singular terms come only from the region of small y. For small x, we have

$$\begin{aligned}
\int \frac{d^6k_i e^{ik_i\cdot x_i}}{(k^2+m^2)} &= \frac{16\pi^3}{x^4} - \frac{4\pi^3 m^2}{x^2} + \frac{1}{2}m^4\pi^3\ln\left(\frac{1}{x^2m^2}\right) + \cdots \\
\int \frac{d^6k_i e^{ik_i\cdot x_i}}{(k^2+m^2)^2} &= \frac{4\pi^3}{x^2} - m^2\pi^3\ln\left(\frac{1}{x^2m^2}\right) + \cdots \\
\int \frac{d^6k_i e^{ik_i\cdot x_i}}{(k^2+m^2)^4} &= \frac{1}{2}\pi^3\ln\left(\frac{1}{x^2m^2}\right) + \cdots\,.
\end{aligned} \tag{4.10}$$

The IR singular terms come from taking the leading term in these three expressions, and we find

$$V_2(p_1,p_2,p_3) = \frac{g^5\pi^3}{2^5}\int^{\Lambda}\frac{d^6y}{(2\pi)^6}\frac{1}{y^4(y+\theta p_2)^2}\ln\left(\frac{1}{(y-\theta p_3)^2}\right). \tag{4.11}$$

The small momentum behavior of this integral when any one or all three momenta are scaled to zero is reproduced by the function

$$\begin{aligned}
\hat{V}_2(p_1,p_2,p_3) = \frac{g^5}{2^{11}}\Bigg(&\frac{1}{2}\ln(p_2\circ p_2)\ln(p_2\circ p_2 + p_3\circ p_3) - \\
&- \frac{1}{4}\ln^2(p_2\circ p_2 + p_3\circ p_3) - \frac{1}{4}\ln(p_2\circ p_2 + p_3\circ p_3)\Bigg).
\end{aligned} \tag{4.12}$$

Note that the logarithmic divergence that arises as $p_2 \to 0$ comes from the three-point subdiagram which would be divergent in the commutative theory. On the other hand, setting p_1 or p_3 to zero does not lead to a divergence. It is possible that there is some other function with the same singularities as (4.12) whose form would be more suggestive of an interpretation by χs.

For ϕ^3 theory with a single ϕ, we should sum the contribution of V_1 with that of V_2 (which is automatically symmetrized in momenta when inserted into the expression (4.7)) to get the complete two loop contribution to the $(\operatorname{tr}\phi)^3$ term in the effective action. In writing a wilsonian action to reproduce the IR singularities of the ϕ^3 theory, we do not necessarily need to match the behavior diagram by diagram; however it is possible to take a theory which isolates the contribution V_1, for example. In the theory with lagrangian density

$$\begin{aligned}
\operatorname{tr}\Bigg(&\frac{1}{2}\sum_i(\partial\phi_i)^2 + \frac{1}{2}m^2\sum_i\phi_i^2 + \\
&+ g(\phi_1\star\phi_4\star\phi_4 + \phi_2\star\phi_5\star\phi_5 + \phi_3\star\phi_6\star\phi_6 + \phi_4\star\phi_5\star\phi_6 + \text{c.c.})\Bigg)\,,
\end{aligned} \tag{4.13}$$

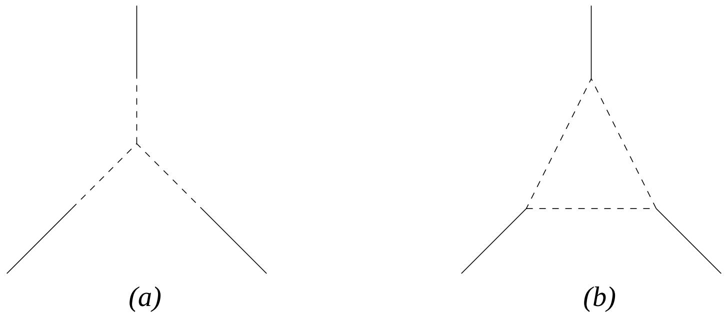

Figure 5: Possibilities for the interpretation of two loop IR singularities.

the leading $\mathrm{tr}(\phi_1)\,\mathrm{tr}(\phi_2)\,\mathrm{tr}(\phi_3)$ term in the effective action is precisely V_1. Thus, we would like to understand how the singularity $\ln(p_1^2 + p_2^2 + p_3^2)$ alone or terms of the form (4.12) could arise from a wilsonian effective action.

4.2 Interpretation of the singularities

The two loop IR singularities we have found have momentum dependence that cannot be reproduced by tree level diagrams with local couplings only on the brane (such as the tree level χ exchange diagrams which reproduced all one loop singularities). Each propagator and vertex factor of such a diagram is a function of a single sum of external momenta, while the singularities we have found, (4.6) and (4.12), cannot be reproduced by a product of such functions.

If our interpretation of the ψ fields as higher dimensional particles is correct, one possibility would be that these singularities arise from a diagram like that of figure 5a, in which each ϕ becomes a ψ and these three ψ field interact locally in the bulk. This is suggested both by the $(\mathrm{tr}\,\phi)^3$ structure and by the fact that viewed as worldsheet diagrams in string theory, the double line versions of the diagrams in figure 4 are topologically equivalent to the closed string interaction diagram in figure 6.

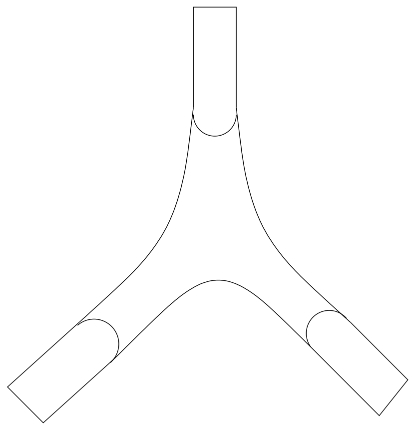

Figure 6: Closed string interaction diagram topologically equivalent to the diagrams of figure 4.

There are various possibilities for the form of a ψ^3 interaction, including possible derivatives on the ψs and the possibility of a coupling that varies as some function of the transverse coordinates $x_\perp$ (for example, a varying dilaton). Though such interactions do seem to give IR singularities with nontrivial dependence on external momenta, we have not found a simple action with local interactions of the ψs in the bulk that reproduces the two loop singularities above.

A second possibility would be that the momentum dependence of the two loop IR singularities arises from a loop of χs (or ψs), such as the diagram of figure 5*b*. The form of the expressions (4.5) and (4.11) are suggestive of such a diagram, if we reinterpret the y integrals as integrals over a loop momentum $l = \theta y$. For example, the expression (4.5) becomes

$$V_1(p_1, p_2, p_3) = \frac{g^5\pi^3}{3\cdot 2^5}\det(\Theta)\int^{\Lambda}\frac{d^6 l}{(2\pi)^6}\frac{1}{l\circ l\ (l+p_2)\circ(l+p_2)\ (l-p_3)\circ(l-p_3)}\,. \tag{4.14}$$

which is exactly reproduce by the diagram of figure 5*b* with a coupling proportional to $g^{5/3}\det{}^{1/3}(\Theta)\phi(x)\chi_0(x)\chi_0(x)$ where χ_0 propagates on the brane with metric $g_{\mu\nu}$. Such a coupling is undesirable, however, since it would lead to higher point functions with fractional powers of the coupling and more severe IR singularities which are not present in the original theory.

To conclude, we do not have a satisfactory interpretation of the two loop IR singularities in terms of weakly coupled light degrees of freedom. We hope to return to this important problem in the near future.

Acknowledgments

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